SIDDHARTH INSTITUTE OF ENGINEERING &TECHNOLOGY:: PUTTUR
ELECTRONICS & COMMUNICATON ENGINEERING

QUESTION BANK (DESCRIPTIVE)

Subject with Code : (18EC0409) PROBABILITY THEORY AND STOCHASTIC PROCESSES

Course & Branch: B. Tech – ECE **Year & Semester:** II & II

Regulation: R18

UNIT-I
INTODUCTION TO PROBABILITY AND RANDOM VARIABLES

1. (a) Define the following with examples. [L1][CO1][5M]
   i. Sample space ii. Event iii. Mutually exclusive events. iv. Independent events.
   (b)Two cards are drawn from a 52 card deck. [L1][CO1][5M]
   i. Given the first card is queen, what is the probability that the second is also a queen?
2. (a) Explain conditional distribution and density function .state its properties [L6][CO1][5M]
   (b) In a bolt factory machine A,B,C Manufacture 30%,30%,40% of the total output respectively. From their outputs,4%,5%,3% are defective bolt.A bolt is drawn at random and found to be defective. What are the probabilities that is was manufactured by machines A, B, C? [L6][CO1][5M]
3. (a) Discuss Joint and conditional probability. [L1][CO1][3M]
   (b) When are two events said to be mutually exclusive? Explain with an example. [L1][CO1][3M]
   (c) Determine the probability of the card being either red or a king when one card is drawn from a regular deck of 52 cards. [L6][CO1][4M]
4. (a) When two dice are thrown, find the probability of getting sum of 10 or 11? [L6][CO1][5M]
   (b) An experiment consists of rolling a single die, two events are defined as, A ={ 6 shown up } B = {2 or 5 shows up } [L6][CO1][5M]
   i. Find P(A) & P(B) ii. Define third event C so that P(c) = 1- P(A)-P(B).
5. (a) State and prove Bayes theorem of probability. [L4][CO1][5M]
   (b) An ordinary 52 Card deck is thoroughly shuffled. You are dealt four cards up. What is the probability that all four cards are fives. [L6][CO1][5M]
6. Define distribution and density function. State its properties. [L1][CO1][5M]
7. (a)Explain the different types of random variables. [L1][CO1][5M]
   (b)Discuss Rayleigh and exponential distribution function. [L1][CO1][5M]
8. (a) Define probability [L1][CO1][5M]
   i. Mathematical approach.
   ii. Relative frequency approach
   iii.set theory approach.
   (b) A die is tossed find the probabilities of the event A = {odd number shows up} , B ={ number larger than 3 } showsup. Find$A \cup B$ and $A \cap B$. [L6][CO1][5M]
9. (a) A shipment of components consists of three identical boxes.one box contains 2000 components of Which 25% Are defective, the second box has 5000 components of which 20% are defective and the Third box contains 2000 components of which 600 are defective. A box is selected at random and a Component is removed at random from the box. Whats the probability that this component is defective? What is the probability that is came from the second box? [L6][CO1][5M]

(b) In a single throw of two dice, what is the probability of obtaining a sum of at least 9. [L6][CO1][2M]

10. (a)State Baye's Theorem. [L4][CO1][2M]

(b)What are the conditions for a function to be a Random variable? [L1][CO1][2M]

(c)What are the conditions to be satisfied for the statistical independence of three events A, B and C. [L1][CO1][2M]

(d)Explain about certainity and uncertainity with suitable examples [L1][CO1][2M]

(e) Define Exhaustive event & mutually exclusive event. [L1][CO1][2M]

UNIT –II

MULTIPLE RANDOM VARIABLES AND OPERATIONS ON MULTIPLE RANDOM VARIABLES

1. (a) Discuss the properties of conditional distribution function. [L4][CO1][5M]

(b) If the joint PDF of two dimensional random variable (x, y) is given by: [L6][CO1][5M]

$$f_{X,Y}(x,y) = 2 \quad ; \quad \text{for } 0\leq X\leq 1,\ 0\leq Y\leq x$$
$$= 0 \quad ; \quad \text{otherwise}$$

Find the marginal density function of X and Y.

2. (a)Random variable X and Y have the density: [L6][CO2][5M]

$$f_{X,Y}(x,y) = 1/24 \quad ; \quad \text{for } 0\leq X\leq 6,\ 0\leq Y\leq 4$$
$$0 \quad ; \quad \text{elsewhere}$$

What is the expected value the function $g(X,Y)=(X,Y)^2$?

(b) Briefly explain about jointly Gaussian random variables. [L1][CO2][5M]

3. The joint pdf is given as $f_{x,y}(x,y) = e^{-(2x+y)}$ for $x \geq 0$ and $y \geq 0$. [L6][CO2][10M]

Find (a) the value of A and

(b) the marginal density functions.

4. (a)Two random variable X and Y with joint density function [L6][CO2][10M]

$$f_{XY}(x,y) = Ae^{-(2x+y)} \qquad x\geq 0,\ \ y\geq 0$$
$$0 \qquad \text{Otherwise}$$

i)Find 'A' ii)Find Marginal density functions?

5. The joint probability density function of two random variables X and Y is given by [L6][CO2][10M]

$$f_{XY}(x,y) = c(2x+y) \qquad 0\leq x\leq 1 \quad 0\leq y\leq 2$$
$$0 \qquad \text{Otherwise}$$

i)Find 'c' ii)Find Marginal density functions?

6. The joint pdf of two random variables X and Y is given by [L6][CO2][10M]

$$f_{X,Y}(x,y) = K(x^2+y^2); \qquad x\geq 0,\ \ y\geq 0$$
$$= 0 \quad ; \qquad \text{otherwise}$$

Find (a) The 'K' value (b) $f_X(x)$ &$f_Y(y)$

7. (a) Define and explain joint distribution function and joint density function of two random variables X and Y. [L1][CO2][5M]

(b) State and prove the properties of joint distribution function. [L4][CO2][5M]

8. Explain conditional distribution and density function –point conditioning and interval conditioning? [L1][CO2][10M]

9 .(a). If the function [L6][CO2][5M]

$$f_{XY}(x,y) = \begin{cases} be^{-2x}\cos(y/2) & 0\leq x\leq \pi \quad 0\leq y\leq \pi \\ 0 & \text{Elsewhere} \end{cases}$$

Where ‘b’ is a positive constant is a valid joint probability density function. Find ‘b’

(b) Explain the sum of two random variables and multiple random variables [L1][CO2][5M]

10.(a).State Central Limit Theorem? [L4][CO2][2M]
(b).Define the expected value of a function of two random variables? [L1][CO2][2M]
(c).How interval conditioning is different from point conditioning. [L4][CO2][2M]
(d).Define joint moments about the origin. [L1][CO2][2M]
(e).Write a brief short note on joint central moments. [L1][CO2][2M]

UNIT –III

RANDOM PROCESS- TEMPORAL CHARACTERISTICS

1.What is ACF? State and explain any four properties of ACF? [L1][CO3][10M]

2.Explain about first order,second,wide-sense and strict sense stationary process. [L1][CO3][10M]

3.(a)Show that the autocorellation function of a stationary random process is an even function of τ. [L4][CO3][5M]
(b)Give the classification of random processes. [L1][CO3][5M]

4. A random process is defined by x(t) =At where A is a continuous random variable uniformly Distributed on (0,1) and t represents time. Find (a) E [x (t)] (b) $R_{xx}[t, t+\tau]$ (c) Is the process stationary? [L6][CO3][10M]

5 .(a) A random process is defined as $X(t)=A\sin(\omega t+\Theta)$,where A is a constant and Θ is a random Variable distributed over $(\pi,-\pi)$,check X(t) is stationary. [L6][CO3][5M]

(b). Prove the following 1. $|R_{XX}(\tau)|\leq R_{XX}(0)$ 2. $R_{XX}(-\tau)= R_{XX}(\tau)$ 3. $R_{XX}(0) = E[X^2(t)]$ [L4][CO3][5M]

6. (a) State the conditions for wide sense stationary random process. [L4][CO3][5M]
(b) Write short notes on ergodic random processes. [L1][CO3][5M]

7. What is cross correlation function of a random process? state and explain any four properties of Cross correlation function of a random process? [L1][CO3][10M]

8.(a) Explain about mean-ergodic process. [L1][CO3][5M]
(b).If x (t) is a stationary random process having mean = 3 and auto correlation function: $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean and variance of the random variable. [L6][CO3][5M]

9.(a) Explain the significance of auto correlation. [L1][CO3][5M]

(b) Find auto correlation function of a random process whose power spectral density is given by $4/(1+(\omega^2/4))$ [L6][CO3][5M]

10.(a).Test the function ''$e^{\tau}u(\tau)$''for a valid PSD. [L4][CO3][2M]

(b).Define WSS random process. [L1][CO3][2M]

(c).What is a stationary process? Explain. [L4][CO3][2M]

(d).Determine the mean square value of a random process with autocorrelation function $R_{XX}(\tau)=e^{-|\tau|}$ [L6][CO3][2M]

(e).Write the condition two WSS process X(t) and Y(t)are jointly wide sense stationary? [L1][CO3][2M]

UNIT –IV

RANDOM PROCESS- SPECTRAL CHARACTERISTICS

1.(a) Briefly explain the concept of cross power density spectrum. [L1][CO4][5M]

(b) Find the cross correlation of functions sin ωt and cosωt. [L6][CO4][5M]

2.(a) The power spectral density of a stationary random process is given by [L6][CO4][5M]

$S_{xx}(\omega) = A; \quad -k < \omega < k$

$\quad 0; \quad$ otherwise

Find the auto correlation function.

(b)Discuss the properties of power spectral density. [L4][CO4][5M]

3. (a)Discuss the properties of cross power density spectrum. [L4][CO4][5M]

(b)Discuss the relation between cross power spectrum and cross correlation function. [L4][CO4][5M]

4.State and prove properties of PDS [L4][CO4][10M]

5. (a) If the PSD of x(t) is $S_{XX}(\omega)$.Find the PSD of dx(t)/dt. [L6][CO4][5M]

(b)Find the PSD of a stationary random process for which autocorrelation is $R_{XX}(\tau)=6e^{-\alpha|\tau|}$ [L6][CO4][5M]

6.(a) State and prove wiener –khintchins relations [L4][CO4][5M]

(b) Prove that 1. $S_{XX}(-\omega)= S_{XX}(\omega)$ 2. $S_{XY}(\omega)= S_{YX}(-\omega)$ [L4][CO4][5M]

7.The psd of X(t) is given by [L6]CO4][10M]

$S_{xx}(\omega) = 1+\omega^2 \quad$ for $|\omega|<1$

$\quad 0 \quad$ otherwise

Find the autocorellation function.

8.The power spectral density of a stationary random process is given by [L6][CO4][10M]

$S_{xx}(\omega)=A \quad -K\leq\omega\leq K$

$\quad 0 \quad$ otherwise

Find the autocorrelation function.

9.(a)A stationary random process X(t) has autocorrelation $R_{XX}(\tau)=10+5\cos(2\tau)+10e^{-2|\tau|}$. Find the dc and ac powers of X(t). [L6][CO4][5M]

(b).Prove that $S_{XX}(\omega)=S_{XX}(-\omega)$ [L4][CO4][5M]

10.(a).Write some properties of auto Power density Spectrum? [L4][CO4][2M]

(b).Derive the power spectral density at zero frequency is equal to the area under the curve of the autocorrelation $Rxx(\tau)$? [L4][CO4][2M] (c).Derive the formula for power spectral density is an even function? [L4][CO4][2M]

(d).Derive the formula for time average of the mean square value of WSS random process is equal to the area under the curve of the power spectral density? [L4][CO4][2M]

(e).Derive the formula for sxy(w)=0 &syx(w)=0,if X(t) and Y(t) are orthogonal? [L4][CO4][2M]

UNIT –V
LINEAR SYSTEMS WITH RANDOM INPUTS

1.(a). Derive the relation between PSDs of input and output random process of an LTI system. [L4][CO5][5M]

(b). Discuss about cross correlation between the input X (t) and output Y (t). [L4][CO5][5M]

2. (a) Explain about LTI system [L1][CO5][5M]

(b) Find the power density spectrum of response of a linear system [L4][CO5][5M]

3. (a)X(t) is a stationary random process with zero mean and auto correlation $R_{xx}(t)=e^{-2I\tau I}$ is applied to a system Of functionH(ω)=1/jω+2.Find mean and PSD of its output. [L6][CO5][5M]

(b)Find the autocorellation of the response Y(t). [L4][CO5][5M]

4. Write notes on: [L1][CO5][10M]

(a) Band Pass random process.

(b) Band limited random process

(c) Narrow band random process.

5. (a) Derive the relation between PSD of input and output random process of an LTI system. [L4][CO5][5M]

(b) Discuss about cross correlation between the input X (t) and output Y (t) [L4][CO5][5M]

6. Derive the expressions for mean. Autocorrelation cross correlation and PSD of response of a linear System [L4][CO5][10M]

7.(a)How mean of the system response Y(t) is calculated? [L4][CO5][5M]

(b)Write different types of band pass processes with band limited processes. [L1][CO5][5M]

8. (a)Define mean value of system response. [L4][CO5][5M]

(b)Find mean square value of Y(t). [L4][CO5][5M]

9.(a) A WSS random process x(t) is applied to the input of an LTI system whose impulse response is $5te^{-2t.}$. The mean of x(t) is 3.Find the mean output of the system [L6][CO5][5M]

(b) Give any two spectral characteristics of the system response. [L1][CO5][5M]

10.(a).Write on a brief note on auto correlation function of output response? [L1][CO5][2M]

(b).Explain mean value of output response. [L1][CO5][2M]

(c).Define a linear system. [L1][CO5][2M]

(d).Define mean square value of output response. [L1][CO5][2M]

(e).Define band pass random processes [L1][CO5][2M]